

Name

Class



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# Functions

(9 – 1) Topic booklet

## Higher

These questions have been collated from previous years GCSE Mathematics papers.

**You must have:** Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser.

Total Marks

### Instructions

- Use **black** ink or ball-point pen.
- Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Answer the questions in the spaces provided  
– *there may be more space than you need.*
- Diagrams are NOT accurately drawn, unless otherwise indicated.
- You must **show all your working out.**
- If the question is a **1H** question you are not allowed to use a calculator.
- If the question is a **2H** or a **3H** question, you may use a calculator to help you answer.

### Information

- The marks for **each** question are shown in brackets  
– *use this as a guide as to how much time to spend on each question.*

### Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end.

**Answer ALL questions**  
**Write your answers in the space provided.**  
**You must write down all the stages in your working.**



9 The functions  $f$  and  $g$  are such that

$$f(x) = 3(x - 4) \text{ and } g(x) = \frac{x}{5} + 1$$

(a) Find the value of  $f(10)$

.....  
(1)

(b) Find  $g^{-1}(x)$

$$g^{-1}(x) = \text{.....}$$

(2)

(c) Show that  $ff(x) = 9x - 48$

(2)

10  $f(x) = 4\sin x^\circ$

(a) Find  $f(23)$

Give your answer correct to 3 significant figures.



.....  
(1)

$g(x) = 2x - 3$

(b) Find  $fg(34)$

Give your answer correct to 3 significant figures.

.....  
(2)

$h(x) = (x + 4)^2$

Ivan needs to solve the following equation  $h(x) = 25$

He writes

$$(x + 4)^2 = 25$$

$$x + 4 = 5$$

$$x = 1$$

This is not fully correct.

(c) Explain why.

.....  
.....  
(1)

**10** The function  $f$  is such that

$$f(x) = 4x - 1$$



(a) Find  $f^{-1}(x)$

$$f^{-1}(x) = \dots\dots\dots (2)$$

The function  $g$  is such that

$$g(x) = kx^2 \text{ where } k \text{ is a constant.}$$

Given that  $fg(2) = 12$

(b) work out the value of  $k$

$$k = \dots\dots\dots (2)$$

Sample 1 – Paper 3H

**(Total for Question 10 is 4 marks)**

**11**  $f$  and  $g$  are functions such that

$$f(x) = \frac{2}{x^2} \quad \text{and} \quad g(x) = 4x^3$$



(a) Find  $f(-5)$

.....  
(1)

(b) Find  $fg(1)$

.....  
(2)

June 2018 – Paper 2H

**(Total for Question 11 is 3 marks)**

**16** The functions  $f$  and  $g$  are given by



$$f(x) = \frac{12}{x+1} \quad \text{and} \quad g(x) = 5 - 3x$$

(a) Find  $f(-3)$

.....  
(1)

(b) Find  $fg(1)$

.....  
(2)

(c) Find  $g^{-1}(4)$

.....  
(2)

**18** The function  $f$  is given by

$$f(x) = 2x^3 - 4$$

(a) Show that  $f^{-1}(50) = 3$

(2)

The functions  $g$  and  $h$  are given by

$$g(x) = x + 2 \quad \text{and} \quad h(x) = x^2$$

(b) Find the values of  $x$  for which

$$hg(x) = 3x^2 + x - 1$$

(4)

**18**  $f(x) = 3x^2 - 2x - 8$



Express  $f(x + 2)$  in the form  $ax^2 + bx$



**19** The functions  $f$  and  $g$  are such that



$$f(x) = (2x + 3)^2 \quad \text{and} \quad g(x) = 2x - 1$$

(a) Find  $gf(-3)$

.....  
(2)

(b) Find  $g^{-1}(x)$

$g^{-1}(x) =$  .....  
(2)

**19** The functions  $g$  and  $h$  are such that



$$g(x) = \sqrt[3]{2x - 5} \qquad h(x) = \frac{1}{x}$$

(a) Find  $g(16)$

.....  
(1)

(b) Find  $hg^{-1}(x)$   
Give your answer in terms of  $x$  in its simplest form.

$hg^{-1}(x) =$ .....  
(3)

**19**  $f$  and  $g$  are functions such that

$$f(x) = \frac{12}{\sqrt{x}} \quad \text{and} \quad g(x) = 3(2x + 1)$$

(a) Find  $g(5)$

.....  
(1)

(b) Find  $gf(9)$

.....  
(2)

(c) Find  $g^{-1}(6)$

.....  
(2)

**19** For all values of  $x$

$$f(x) = (x + 1)^2 \quad \text{and} \quad g(x) = 2(x - 1)$$

(a) Show that  $gf(x) = 2x(x + 2)$

(2)

(b) Find  $g^{-1}(7)$

.....  
(2)

**20** For  $x \geq 0$ , the functions  $f$  and  $g$  are such that

$$f(x) = 3x + 4 \qquad g(x) = \frac{\sqrt{x} + 2}{5}$$

(a) Find  $g^{-1}(x)$

$$g^{-1}(x) = \dots\dots\dots (2)$$

(b) Solve  $gf(x) = 3$

$$x = \dots\dots\dots (3)$$

**21** The functions  $f$  and  $g$  are such that

$$f(x) = 3x^2 + 1 \quad \text{for } x > 0 \quad \text{and} \quad g(x) = \frac{4}{x^2} \quad \text{for } x > 0$$

(a) Work out  $gf(1)$

.....  
(2)

The function  $h$  is such that  $h = (fg)^{-1}$

(b) Find  $h(x)$

.....  
(4)

**21** The functions  $f$  and  $g$  are such that

$$f(x) = 3x - 1 \quad \text{and} \quad g(x) = x^2 + 4$$

(a) Find  $f^{-1}(x)$

$$f^{-1}(x) = \dots\dots\dots (2)$$

Given that  $fg(x) = 2gf(x)$ ,

(b) show that  $15x^2 - 12x - 1 = 0$

(5)

**22**  $f(x) = \sqrt[3]{x}$   
 $g(x) = 2x + 3$

$h(x) = fg(x)$

Find  $h^{-1}(x)$



$h^{-1}(x) = \dots\dots\dots$

November 2022 – Paper 2H

**(Total for Question 22 is 3 marks)**



**22** The functions  $f$  and  $g$  are such that

$$f(x) = 5x + 3 \quad g(x) = ax + b \quad \text{where } a \text{ and } b \text{ are constants.}$$

$$g(3) = 20 \quad \text{and} \quad f^{-1}(33) = g(1)$$

Find the value of  $a$  and the value of  $b$ .



$$a = \dots\dots\dots$$

$$b = \dots\dots\dots$$